A Practical Introduction to

APL 1
&
APL 2

by

Graeme Donald Robertson

. . TRAINING THAT WORKS . . .

Date: .....

Place: .....

Instructor: .....

Student(s): .....

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Conduct of this 2 day course:

After short introductions, the student group is invited to divide up into pairs.
Each pair works on one computer/terminal for the duration of the course.
Each pair is given the first lesson and asked to work through it on their computer at their own pace.
Pairs are encouraged to help each other with new concepts and difficulties as they arise and to experiment on the computer with any ideas which they think they can express in APL statements.
Tuition is given when problems cannot be resolved by the pair. Questions may be answered directly on matters of fact, or indirectly by way of a suggestion as to how the problem might be tackled.
Each day covers about 7 lessons, depending upon the pace of each pair.
There is no pressure to complete all lessons (remaining notes are given out at the end of the course).
At the discretion of the tutor, lessons may be skipped or assigned for private study after the course has ended.
Short synopses are given (with an overhead projector or white board) at suitable intervals throughout the course to the group as a whole.

A Practical Introduction to

APL 1&2

APL is the only language to have been 200 years in the debugging.
Ken Iverson

Day 1: First Generation – APL 1

APL 1 - Core APL

Session 1

- APL Character Set
- APL Keyboard
- Primitive Functions
- Simple Arrays
- Assignment of Variables
- Indexing & Special Syntax
- Error Messages

Session 2

- User-Defined Functions
- Editing Functions
- Local & Global Variables
- Order of Execution of Functions
- System Commands
- APL Idioms

...learning by practice, by induction, and by heuristic methods...
...pragmatic teaching by encouraging experiment and by individual tutorial.
Ken Iverson

LESSON 0

Why Learn APL?

APL is a high-level, general-purpose, *intuitive programming language* which is designed to be easy on the programmer even if consequently hard on the computer - through *power*, not inefficiency.

APL has its own special character set of around 200 alphabetic characters and symbols. Although the APL symbols might appear illegible and unintelligible, each individual symbol performs a specific task making programs very concise. APL is A Programming Language which is essentially *simple and easy to learn*, and APL is interactive making it possible to *experiment with different ideas* while developing solutions.

Getting to Know Your APL Keyboard

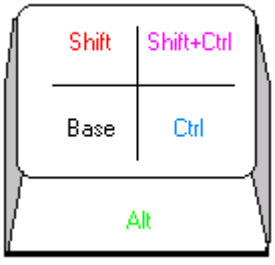
Your computer should be set up already so that an APL session is visible and has the focus. Typing on your keyboard should cause characters to be displayed on your screen. Try typing something. When you come across a new symbol, or key combination, write it on the supplied blank keyboard. This will help you quickly to become familiar with the new APL key layout.

Symbols which require the *Shift* key to be pressed should be written in the upper left hand corner of the corresponding key cap on the supplied blank keyboard chart. Symbols which require the *Ctrl and Shift* keys to be pressed should be written in the upper right hand corner of the corresponding key on the chart. Symbols which require the *Alt* key to be pressed should be written on the front of the corresponding key, as shown below. (Beware of *Ctrl* keys on a mainframe.)

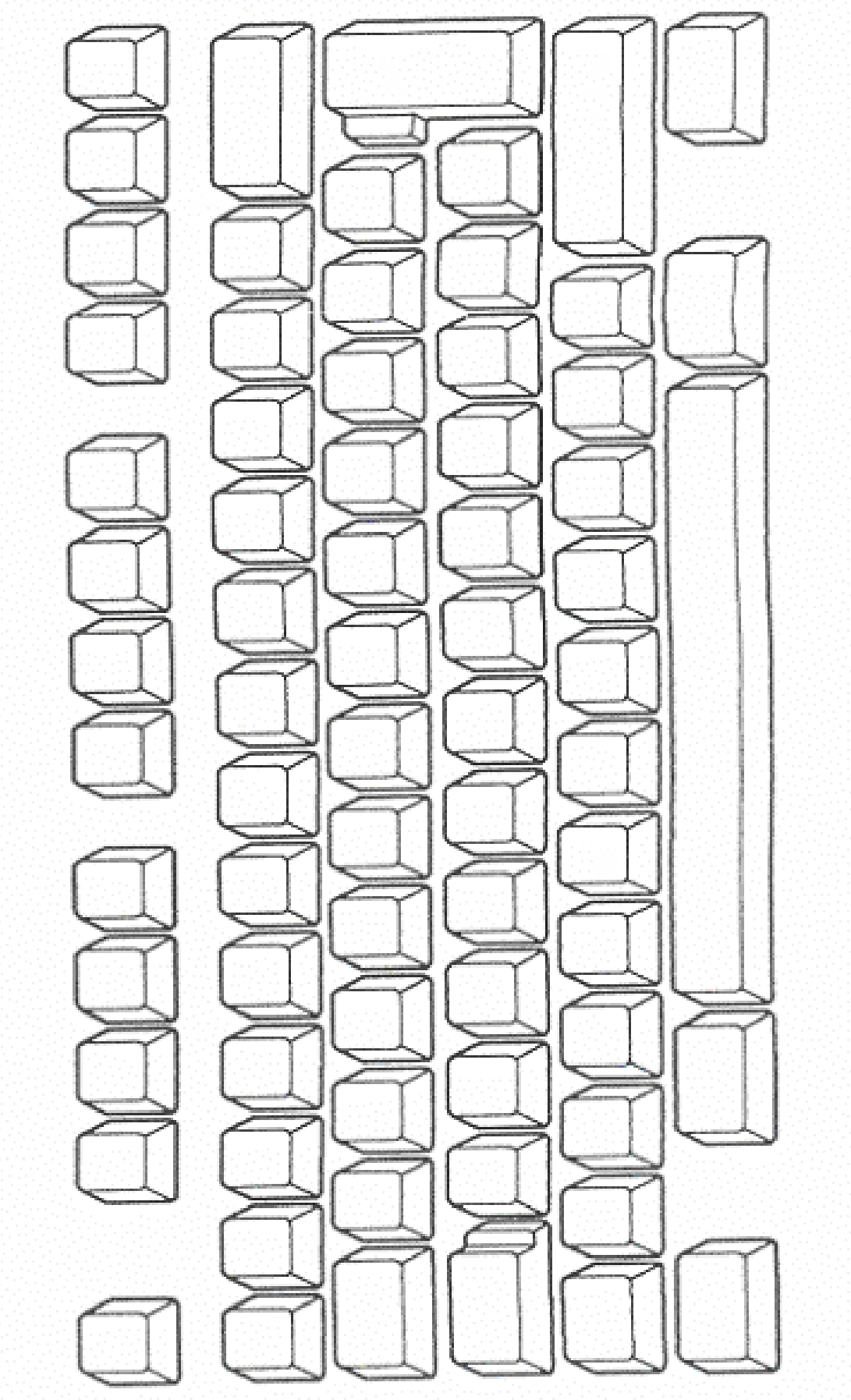
- Type the numbers 0 to 9 on your keyboard and write them in the lower left hand corner of the corresponding key on your keyboard chart. Type in the upper case letters A to Z on your keyboard and write them in the appropriate positions on the keyboard chart.

- Find the symbols + and − on the keyboard and write them on your chart. Find the symbols × and ÷ on the keyboard and write them on your chart. Use the backspace key to rub out the typing. Put that key on your chart too.

- In APL each expression which is typed into the APL session is executed when the Enter key is pressed. In mainframe APL2 the key which is used to enter expressions is the right Ctrl key, and possibly also the numeric pad Enter key. Mark the appropriate key on your chart.



- Ask your tutor for LESSON 1.



LESSON 1

Simple Arithmetic Expressions

Go at your own pace. Experiment. Try to work it out. Think. Talk about it.

- Use APL to add any two numbers together. Check the result. For example, type

```
      65.35 + 35.65
101
```

*Hint: Hit the **Enter** key when you are ready to execute the line containing the cursor.*

Notice how, in *immediate execution mode*, APL indents the cursor 6 spaces to indicate that it is ready to accept the next line of user input. Everything which has been input by the user is indented by 6 spaces, and is **coloured green** in mainframe APL2. Output from the computer starts at the left hand margin and is **coloured red**, as are error lines.

- Type the following two lines into your session and explain the results.

```
      14 - 9
5
      - 7
¯7
```

Notice the distinction between the **negate** function (-) and the **negative** sign**,** or high minus symbol, (¯), which is an intrinsic part of a number, like the decimal point.

Symbols such as - and + can be used either with a *right argument* (which is called the *monadic* or prefix case) or with a *left and right argument* (which is called the *dyadic* or infix case). Thus the hyphen symbol can be used monadically to mean negate or dyadically to mean **minus,** or subtract. Write the new high minus symbol on your keyboard chart.

The **plus** and **minus** signs were introduced by the German mathematician Johann Widmann in 1489 to signify addition and subtraction. Dyadic deployment of the symbols is now familiar to everyone.

APL has many such powerful primitive functions which allow complex computations to be done very easily. Primitive functions follow the principle of one symbol per mathematical operation.

- Experiment to see if you can deduce the monadic and dyadic meanings of the symbols × ÷ | ⌈ ⌊ * ⍟ ! both by applying simple numeric arguments, and by inference from the form of the symbol itself.

The **times** sign was introduced by the English mathematician who invented the slide rule; William Oughtred (1575 - 1660). Its use to signify multiplication is now familiar to everyone who has been exposed to the language of elementary algebra. Most computer languages use * to indicate multiplication. Algebraists use a variety of alternative ways to indicate multiplication: a×b or a.b or ab. APL consistently uses ×. "APL is the only [computer] language to have been two hundred years in the debugging," says Iverson.

APL is derived from mathematical notation. It did not appear from the standard evolutionary origins of most other computer languages. APL crystallized from an unconstrained theoretical notation (*Iverson notation*) when it was realized that it could be executed on a computer. "I wasn't trying to design or implement a language for a machine," confessed Iverson.

The monadic meanings of × ÷ | ⌈ ⌊ * ⍟ and ! are **direction**, **reciprocal**, **magnitude**, **ceiling**, **floor**, **exponential**, **natural logarithm** and **factorial** respectively, and dyadic meanings **multiply**, **divide**, **residue**, **maximum**, **minimum**, **power**, **logarithm** and **binomial** respectively.

- Investigate the monadic meaning of + Experiment with any suggestive arguments! ;-)

The *result* of one expression can be used as the argument to another function.

- Try some compound expressions such as

```
      3 × 4 + 6
30
```

and

```
      (3 × 4) + 6
18
```

Hence explain the result of the expression

```
      14 - 6 - 5 - 3 - 7
17
```

Beyond BIDMAS. Remember BIDMAS (or BODMAS)? It tells you the order of precedence in simple arithmetic expressions – brackets first, then indices (or of), division, multiplication, addition and finally subtraction. APL, on the other hand, does not assume any special order of precedence between functions. Execution simply proceeds from right to left unless you use **parentheses** (round brackets) to control the order of execution. All APL functions have equal priority. This basic "right-to-left" grammatical rule applies to dyadic functions and monadic functions alike in APL. (It's a bit like the rule in English that the object of a sentence comes after the verb.)

Rule 1: The right argument of any function, monadic or dyadic, is the result of the entire expression immediately to its right.

Some functions take boolean arguments and return boolean results.

- Reading 1 as true and 0 as false, verify the truth values of these expressions.

```
      1 ∨ 1
1
      1 ∧ (0 ∧ 1) ∨ 1 ∨ 0
1
      ~ 0 ∨ ~ 0
0
```

These invoke the simple logical functions: **and** (∧), **or** (∨) and **not** (~).

- Some functions take numeric arguments and return boolean results. Verify the results of

```
      20.5 = 41 ÷ 2
1
      101 < 200 - 100
0
      27.3 > 39.31
0
```

These introduce binary relational functions: **equals** (=), **less-than** (<) and **greater-than** (>).

- Trigonometric functions are implemented via the dyadic **circle** function. A left argument of 1 returns the sine of the right argument. Assess the result of

```
      1 ○ 3.14159
0.000002654
```

knowing that Sin(π) is zero. A left argument of 2 returns the cosine, 3 returns the tangent.

The left argument of the circle function may be an integer between 12 and -12 representing various standard pythagorean, trigonometric, hyperbolic and complex number functions.

- Explore a few examples. Find the meaning of the monadic circle function.

- Type the following line into your session and execute it a few times.

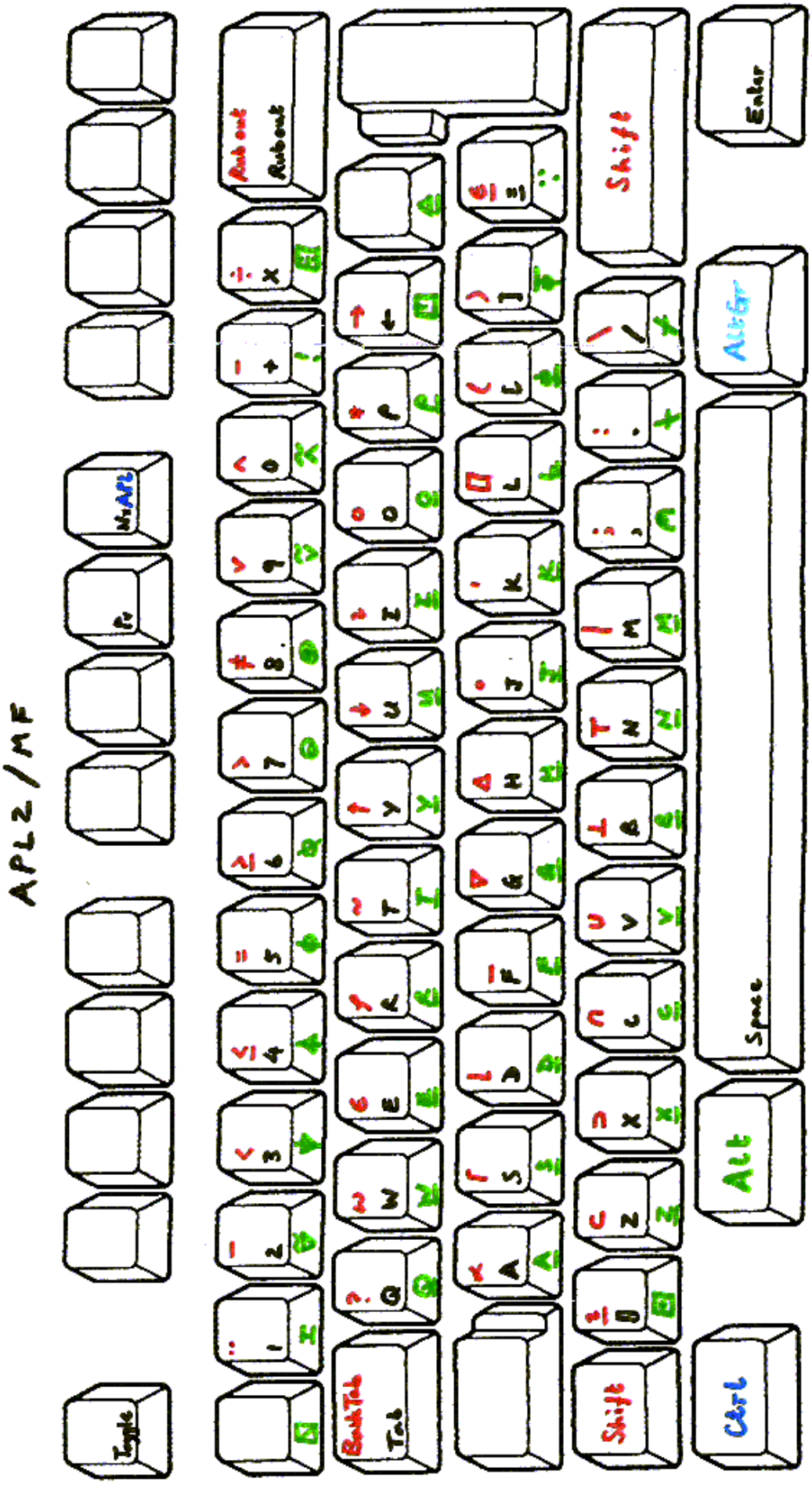
```
      ? 6
5
```

What do you think the results indicate about the meaning of roll (?)? ☺

- Try some more adventurous examples of the application of Rule 1.

```
      ⌊0.5+3.23
3
      ⌈¯0.5+3.23
3
      ⌊¯3.5⌈¯8.2
4
      3.2|¯4.3
2.1
      ((?5)≤?8)∨(?5)≥?8
1
      (~1⍲0⍱1)=1≤0.2≤3≤4
0
      (2!6)=(!6)÷(!2)×!4
1
      ((3×4)=⋆(⍟3)+⍟4)∧(3÷4)=⋆(⍟3)-⍟4
1
```

- Ask your tutor for LESSON 2.


APL2/MF
Toggle
Alt Out
Autocut
BackTab
Tab
Shift
Shift
Enter
Space
Alt
AltGr
Ctrl

LESSON 2

Names, Lists & Literals

Numbers and results of expressions can be assigned to names. The left assignment arrow (←) may be read as **gets** or **is-assigned** or simply **is**.

- Enter the following statements (or sentences), noting that ⋆ means power and × means times,

```
      INTEREST ← 0.09
      YEARS ← 6
      VALUE←500×(1+INTEREST)⋆YEARS
```

- Type in the name of a variable and hit the Enter key to display the contents of the variable in the session.

```
      VALUE
838.6
```

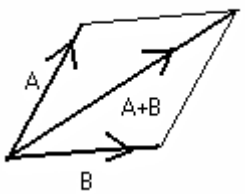
Generalized Scalar Functions. Some functions that take single numbers (**scalar**) arguments have well defined behaviour when the arguments are extended to lists of numbers (**vector**).

- Execute the line

```
      1 2 3 + 4 6 8
5 8 11
```

and explain the result.

Addition of vectors is not a new concept. Newtonian mechanics (c. 1687) employs 3 element vectors to describe positions, velocities and forces in 3D space. Addition of forces may be represented
by lines and parallelograms, as below. The sum is calculated by *vector addition*, using the **plus** sign, as above.



N-dimensional vector spaces (containing N-element vectors) are now employed routinely in many branches of pure and applied mathematics. Indeed the concept of a vector space is one of the principal unifying concepts in the whole of mathematics (see Hilbert Space in Wikipedia).

"The use of a programming language in which elementary operations are extended systematically to arrays provides a wealth of useful identities," says Dr Kenneth Iverson in his book *A Programming Language, Wiley 1962*.

APL adopts this element-by-element approach to vector addition and generalises it to many standard mathematical functions, taking dyadic **plus** and monadic **negate** as role models, or templates.

- Check the results of

```
      - 3 4 5
¯3 ¯4 ¯5
      - 3 4 ¯5 4 6 5.0 ¯8.567
¯3 ¯4 5 ¯4 ¯6 ¯5 8.567
      1 2 3 × 2 ¯2 2
2 ¯4 6
      45 ¯3 2 2.33 + 99 7 4 0.4
144 4 6 2.73
```

- Explore other expressions, using lists of numbers as arguments to the primitive scalar functions represented by symbols + - × ÷ | ⌈ ⌊ * ⍟ ○ !

Scalar Extension. If one of the arguments of a scalar dyadic function is a scalar and the other is a vector (or list) then the scalar is automatically extended to have the same length as the vector.

- Enter

```
      1 ○ .1 .2 .3
0.09983 0.1987 0.2955
```

- Compare with

```
      1 1 1 ○ .1 .2 .3
0.09983 0.1987 0.2955
```

and

```
      1 2 3 ○ .1 .2 .3
0.09983 0.9801 0.3093
```

Otherwise, if the arguments have incompatible lengths then a *LENGTH ERROR* is reported.

- Try to execute the following line.

```
      1 2 ○ .1 .2 .3
```

Literals. Variables can be assigned to lists of literal characters as well as to lists of numbers. Character strings have to be enclosed inside APL *quotes* in order to distinguish literal characters from defined names or simple numerics.

- Enter your name and web address, e.g.

```
      NAME←'DEBBIE ROBERTSON'
      ADDRESS←'APL4.NET'
```

The dyadic structural functions **catenate** (,) **take** (↑) and **drop** (↓), and the monadic structural function **reverse** (⌽), can be used on any list of numbers or characters to produce a new related list.

- Try

```
      ⌽NAME
NOSTREBOR EIBBED
      7↑NAME
DEBBIE
      E←(6↑NAME),'@',ADDRESS
      E
DEBBIE@APL4.NET
```

- Monadic use of Greek letter rho (function **shape**) returns the number of elements in the vector *NAME*. Check the result of

```
      ⍴NAME
16
```

- Dyadic rho (**reshape**) returns the right argument reshaped to have exactly the number of elements specified by the left argument. Try

```
      4⍴ADDRESS
APL4
      40⍴ADDRESS
APL4.NETAPL4.NETAPL4.NETAPL4.NETAPL4.NET
      ⌽50⍴NAME,' '
NOSTREBOR EIBBED NOSTREBOR EIBBED NOSTREBOR EIBBED
```

The shape of a vector is the number of elements in the list.

- Type

```
      ⍴NUMS←56 87 75 80 79 86 84 90
8
      ⍴CHARS←'56 87 75 80 79 86 84 90'
23
```

Literal digits can be converted into numbers using the very powerful **execute** (⍎) function (which is said to make APL 'self-conscious') and numbers can be converted into characters using the very useful **format** (⍕) function.

- Explain the results of

```
      ⍴⍕NUMS
23
      CHARS=⍕NUMS
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

```
      ⍴⍎CHARS
8
      NUMS=⍎CHARS
1 1 1 1 1 1 1 1
      CHARS,⍕NUMS
56 87 75 80 79 86 84 9056 87 75 80 79 86 84 90

      ⍎(3×5)⍴'NUMS◇'
56 87 75 80 79 86 84 90
56 87 75 80 79 86 84 90
56 87 75 80 79 86 84 90
```

The *diamond* symbol (◇) is not a function. It is a statement separator. You might not be able to find it on your APL2 mainframe keyboard. However, diamond is an example of an overstruck character – from the days when space for characters was scarce. A diamond can be input using the three consecutive symbols <_> where _ is the printable backspace. This requires that you first type the command

```
      )PBS ON
```

in APL2, or switch to replace mode via the Insert key in Dyalog APL.

Interval (⍳) can be used to generate any uniformly spaced range of numbers.

The monadic meaning of the iota character (⍳) is a function called **interval** or index generator. It takes a scalar argument and returns a vector result.

- Try

```
      ⍳9
1 2 3 4 5 6 7 8 9
      ⍳19
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
      ¯1+(⍳19)÷10
¯0.9 ¯0.8 ¯0.7 ¯0.6 ¯0.5 ¯0.4 ¯0.3 ¯0.2 ¯0.1 0 0.1 0.2 0.3 0.4 0.
5
      0.6 0.7 0.8 0.9
```

- Experiment with examples like

```
      ¯0.3×8-⍳15
¯2.1 ¯1.8 ¯1.5 ¯1.2 ¯0.9 ¯0.6 ¯0.3 0 0.3 0.6 0.9 1.2 1.5 1.8 2.1
```

- Now try

```
      ¯1+(⍳99)÷50
```

or

```
      ¯1+(⍳9999)÷5000
```

- Use the appropriate keystroke (usually *Ctrl+C* on a mainframe keyboard or *Ctrl+Break* on a PC) to interrupt execution of lengthy or verbose operations. Write this important key combination on your keyboard chart. Learn to interrupt without compunction. Waiting for a rogue function to finish may be *very* expensive on a mainframe. You control the computer now.

APL primitive functions appear *atomic* in the sense that they never stop half way through. They either finish completely or appear not to have started at all. Therefore breaking an APL process always leaves the processing stack at a definite given point in an APL program.

APL *idioms* are commonly used combinations of tokens. They are phrases that are immediately recognised by APL programmers when reading APL code. A simple example of an idiom is

```
      ⍳⍴NUMS
1 2 3 4 5 6 7 8
```

which returns a count of the elements in the vector `NUMS`.

- Propose a use for this idiom:

```
      (1↓NUMS)-(¯1↓NUMS)
31 ¯12 5 ¯1 7 ¯2 6
```

Note the occasional judicious use of redundant parentheses to enhance readability.

- Experiment with dyadic functions ⍴ ↑ and ↓ using *scalar* (single) integer left arguments and *vector* (list) numeric (or character) right arguments. Note, in particular, the **shape** of the arguments and the shape of the results.

- Write an expression which rounds a number of pennies to the nearest 12p. Andrew, James, Charles and Marcus each have a building society account: these contain £5,081.09, £11,954.55, £812.97 and £6,241.00 respectively. Each account has a different annual interest rate: 4.1%, 3.5%, 2.6% and 3.25%. Write an expression which returns the interest on each account. Write another expression which returns how much each person could have at the end of ten years of saving, to the nearest 12p?

LESSON 3

Indexing Non-Scalar Arrays

In a simple and intuitive manner, square **brackets** [ ] are used to select items from a list.

- Enter

```
      NUMS[3 2]
75 87
      (⍳99)[NUMS]
44 13 25 20 21 14 16 10
      (33 44 55 66)[3 2 3 3 1]
55 44 55 55 33
      'scarlet'[1 6 2 4 6 7]
secret
```

The **shape** of the result is the shape of the **index**. If no index is included within the brackets (elided index) then the whole vector is returned. e.g.

```
      NUMS[]
87 75 80 79 86 84 90
```

- Use bracket indexing to select the smallest and largest elements from the vector

```
      A ← ? 100 ⍴ 1000
```

*Hint: Monadic **grade-up** (⍋), applied to argument A returns the permutation vector which would sort A in ascending order.*

Matrices. We have generalised the arguments of functions from *scalars* to *vectors*, or lists. Now we generalise further to *matrices*, or tables.

When the concept of a matrix of numbers is first encountered in mathematics it can appear quite forbidding, but they have many uses. For example, they form the bases of representations of continuous groups which have many deep applications in science. We here consider a matrix simply as a rectangular table of numbers or characters.

In order to create a vector we may use the dyadic **reshape** function (⍴), with a *single* numeric left argument, to produce a list of that length containing elements taken consecutively from the right argument. In order to create a matrix we may use the reshape function with a *two element* numeric vector left argument to produce a table which has that number of rows and columns.

- Examine the displayed output from

```
      3 4 ⍴ 999
999 999 999 999
999 999 999 999
999 999 999 999
```

```
      5 5 ⍴ ⍳ 25
 1  2  3  4  5
 6  7  8  9 10
11 12 13 14 15
16 17 18 19 20
21 22 23 24 25
      2 13 ⍴ 'ABCDEFGHIJKLMNOPQRSTUVWXYZ'
ABCDEFGHIJKLM
NOPQRSTUVWXYZ
```

- Create a character matrix called MONTHS that has 12 rows and 9 columns formed from the carefully spaced character string

```
      'JANUARY   FEBRUARY MARCH     APRIL .. DECEMBER '
```

- Use bracket indexing to select rows 3 and 4 and columns 1 and 2.

```
      MONTHS[ 3 4 ; 1 2 ]
MA
AP
```

Note the semicolon to separate dimensions.

In APL, there are often (always?) many ways to accomplish the same task (although some solutions are manifestly more elegant than others).

- Use take and/or drop with 2 element left arguments to produce from *MONTHS* the selection:

```
      MONTHS[ 3 4 9 ; 5 1 2 ]
HMA
LAP
ESE
```

- From the result of

```
    (12↑0 0 1)⌿MONTHS
MARCH
```

deduce the meaning of **compress-first** (⌿) with boolean left argument. How could you use this function and **compress** (/) to make the above selections? Why was compress renamed **replicate** when the left argument was generalized to include integers rather than just booleans?

*Suggestion: Look up **replicate** (/) in your reference manual or help file.*

- Create a variable called *SALES* which has 12 rows and 3 columns and is filled with random numbers between 1 and 1000.

- Select the first row of *SALES* (and all the columns - by eliding the columns entry).

```
      SALES[1;]
935 384 520
```

- Select the third and first column (and all the rows - by eliding the rows entry).

```
      SALES[;3 1]
520 935
 54 831
  8 530
      .
      .
      .
```

Structure functions. The primitive selection functions, which were used for manipulating lists, all generalise to matrices. We have already seen **take** (↑) and **drop** (↓) applied to matrices. There are other primitive functions that change the structure of their arguments.

- Experiment with monadic transpose (⍉) on *SALES* and other matrices.

```
      ⍉SALES
935 831 530 384 687 847 654 911  48 633 366 723
384  35 672  67 589 527 416 763 737 757 248 754
520  54   8 418 931  92 702 263 329 992 983 652
```

- Experiment with **reverse** (⌽) on *SALES* and other vectors and matrices.

```
      ⌽SALES
      ⍉⌽⍉SALES
```

- Look up **take**, **drop** and **reverse-first** (⊖) in your reference manual or help file. Become familiar with your sources of reference.

- Make a report.

```
      REPORT←MONTHS,⍪SALES
```

- Explain the **expand** function (\) and use it to double the width of the report.

```
      ((2×¯1↑⍴REPORT)⍴1 0)\REPORT
JANUARY    935 385 204
FEBRUARY   831 35  54
MARCH      530 672 8
APRIL      384 67  418
MAY        687 589 931
JUNE       847 527 92
JULY       654 416 702
AUGUST     911 763 263
SEPTEMBER  48  737 329
OCTOBER    633 757 992
NOVEMBER   366 248 983
DECEMBER   723 754 652
```

Why did the name of this function not change when it was generalized to allow integer left arguments?

- Try

```
      3 1 0 0 5 1\⍳4
1 1 1 2 0 0 3 3 3 3 3 4
```

Addition of Matrices. The primitive scalar functions generalise to work with n-dimensional arrays (vectors, matrices, 3D arrays, …).

- Try

```
      SALES+SALES
```

- Scalar extension still applies. Try

```
      SALES × 2
```

Higher Rank Arrays. The *rank* of an array is the **shape** of the **shape** of the array. This is a new idiom which gives the dimensionality (or rank) of the array in question; 1 for a vector, 2 for a matrix, 3 for a 3D array, etc...

- Type in

```
      ⍴⍴SPACE ← 2 3 4⍴⍳2×3×4
3
      SPACE
 1  2  3  4
 5  6  7  8
 9 10 11 12

13 14 15 16
17 18 19 20
21 22 23 24
```

Notice how the first plane,

```
      SPACE[1;;]
1  2  3  4
5  6  7  8
9 10 11 12
```

is printed first, followed by a gap and then the second plane

```
      SPACE[2;;]
13 14 15 16
17 18 19 20
21 22 23 24
```

- Display a rank 4 array such as the one below. Notice the 2 blocks, each containing 3 planes, each containing 4 rows and 5 columns, with spacing to reflect this structure.

```
      2 3 4 5⍴0
```

- **Ravel** (,) it.

```
      ,2 3 4 5⍴0
0 0 0 
```

LESSON 4

APL Programming

A name may be assigned to the result of an expression, and is then called a *variable*. A name may be associated with an expression itself, and is then called a user-defined *function*, or program, which can be applied to a variety of different *arguments*. A function executes some action on its argument(s) to produce a result which may be an argument to a further function.

Here is a very simple example of a user defined function which does nothing more than add its *left argument*, which has been named L, to its *right argument*, which has been named R, and return the *result*, which has been named T for total.

```
      ∇ T←L MYPLUS R
[1]     T←L+R
[2]   ∇
```

- To start the editor, type the del symbol (∇) followed by the *header line* followed by Enter.

```
∇ T←L MYPLUS R
```

This opens the editor and prompts you to type a *statement* on line 1 by printing the first line number in brackets followed by some spaces.

```
[1]
```

- Type `T←L+R` and hit Enter. A prompt for the next line is issued.

```
[2]
```

Any number of lines can be added to a single function. Execution flows from one line to the next, consecutively by default. Sometimes functions can have hundreds of lines, sometimes only one. When all the statements needed to build the result of a function have been entered, the function definition may be terminated by entering a closing del (∇). The user then exits the editor and returns to *immediate execution mode*.

```
[2]  ∇
```

Depending upon the particular version of APL being used, the editor may be either the basic *line editor* (available in most versions of APL) or a *full-screen editor* with many built-in WYSIWYG features. In order to use the APL2 full-screen editor you should execute the system command

```
)EDITOR 2
```

Investigate the facilities of best APL function editor on your APL system and use it to define a new copy, `MYPLUS2`. Test the function with a variety of arguments. Explain the error messages.

```
      3 MYPLUS2 4 5 6 7
7 8 9 10
      3 4 MYPLUS2 5 6 7
      MYPLUS2 5 6 7
      3 MYPLUS2 '5 6 7'
```

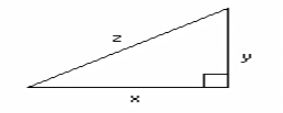
- Incorporate lines such as

```
      M←(⍴L)⌈(⍴R)
      T←(M↑L)+(M↑R)
```

in order to dispense with the first error. Alternatively, consider `T←(M⍴L)+(M⍴R)`

- Write a function `SQT` which takes a numeric right argument and returns its square root.

- Write another function `HYP` which takes, respectively, a left argument and a right argument of the vertical and horizontal sides of a right-angled triangle, and returns the length of the hypotenuse, by Pythagoras' theorem. Does it generalize easily to two or more triangles?



Note: $z = \sqrt{(x^2+y^2)}$ *and you can call* `SQT` *inside* `HYP`.

- Define a function that will return the number of seconds between two times in numeric HOUR, MINUTE, SECOND format. Experiment with **encode** (⊤) and **decode** (⊥). This is useful!

Hint: (2 hrs 14 mins 38 secs) take away (17 mins 9 secs) is (1 hr 57 mins 29 secs) because

```
      0 60 60⊤(0 60 60⊥2 14 38)-0 60 60⊥0 17 9
1 57 29
```

- Use your expression from Lesson 3 to generate a product B report and make it the basis for a function which takes an argument of product name and returns the report for that product.

- Write a function `UNIQUE` that returns a vector of unique numbers in ascending order. e.g.

```
      UNIQUE 1 7 4 7 2
1 2 4 7
```

How would you accommodate character vector arguments?

Hint: Consider `((⍳⍴X)∈X⍳X)/X` *first and sorting last.*

Branching & Flow Control. Within a user-defined function it is possible to jump to some arbitrary line number by means of the **branch** function (→). This function takes a line number as its right argument and does not return a result, but jumps to the line given by the argument. Thus →3 will cause the interpreter to process the statement in line 3 next. This is the basis of flow control in core APL.

- Write a function *ODD* which takes an integer argument and returns a 1 if the argument is odd and a 0 if the argument is even. Do this with and without branching. What about negative numbers?

```
      ∇ BOOL←ODD INT
[1]     P←⌈INT÷2
[2]     Q←⌊INT÷2
[3]     →(6 4)[1+P=Q]
[4]     BOOL←1
[5]     →0
[6]     BOOL←0
      ∇

      ∇ BOOL←ODD INT
[1]     P←⌈INT÷2
[2]     Q←⌊INT÷2
[3]    BOOL←P≠Q
      ∇
```

- Use your reference materials to investigate the definitions of *line labels* and *localised variables* in function definition.

```
      ∇ BOOL←ODD INT;P;Q
[1]     P←⌈INT÷2
[2]     Q←⌊INT÷2
[3]     →(FALSE TRUE)[1+P=Q]
[4]    FALSE:BOOL←1
[5]     →0
[6]    TRUE:BOOL←0
      ∇
```

- Using a loop, write a function which returns the sum total of all the elements in its numeric vector argument.

```
      ∇ TOT←SUM VEC;COUNT;N
[1]     TOT←COUNT←0
[2]     N←⍴VEC
[3]    LOOP:→(COUNT=N)⍴0
[4]     COUNT←COUNT+1
[4]     TOT←TOT+VEC[COUNT]
[5]     →LOOP
      ∇
```

LESSON 5

Revising Functions

To change a function which already exists, enter del (∇), followed by the name of a function. It is then presented for editing. (Note that a closing del is required to exit line edit mode.)

```
∇ MYPLUS
```

- Explore the facilities of your recommended APL function editor. Change the function *MYPLUS*, as you feel appropriate, to make a new function *MYTIMES*.

- Modify the function *HYP* to take a left argument and a right argument of the (x, y) coordinates of the start and end of a line, and return the length of the line by Pythagoras' theorem.

  $z = \sqrt{((x_1-x_2)^2 + (y_1-y_2)^2)}$

- Check that *HYP* gives the correct result for a 3-4-5 triangle and suggest an alternative syntax.

```
0 3 HYP ¯4 0
```

Line 0 Syntax. The *header line*, or line [0], of a user-defined function specifies the calling syntax for the function. The structure of this line determines whether the function is dyadic, monadic or *niladic* (no arguments). It also determines whether the function returns a result.

Quad Input. Quad (⎕) is a variable which communicates between the user's terminal and APL. When ⎕ is referenced, a prompt (⎕:) is displayed indicating that input is being requested.

- For example, type

```
      X ← ⎕          Enter this
⎕:                   System prompts with this
      3              User types in this
      X              Enter this
3                    X has value 3
```

- Write a niladic function called *ASK* which does not return a result. This function should ask the user to type in his gas and electricity meter readings and then ask the user to type in the cost, in pence, of one unit of gas and electricity. The function should then print out the total bill *rounded* to the nearest penny. The session might appear like this:

```
      ASK
What are your Gas and Electricity meter readings?
⎕:
      346 1537
What are your Gas and Electricity unit costs in pence?
⎕:
      0.132 0.0977
Your total energy bill is £195.84p
```

LESSON 6

Some Useful System Commands

System commands are all introduced by an 'unlikely' first character; right parenthesis. This would clearly cause an error in *any* APL expression and therefore causes no conflict or confusion between calls to the operating system and well formed lines of APL code which may be incorporated in functions.

- In order to store on file the variables and functions which you have written, type

```
)SAVE MYWORK
```

This will save a snapshot of your *workspace* in a file called MYWORK on disk.

- In order to reinstate this workspace at any time, use the command

```
)LOAD MYWORK
```

- Other useful system commands include

```
)VARS
```

which lists the global variables that you have assigned, and

```
)FNS
```

which lists the functions that you have defined in the current workspace.

Other important system commands include

```
)SI

)RESET

)ERASE ...

)COPY ...

)WSID

)CLEAR
```

and, last but not least, (because you may need it at the end of the day)

```
)OFF
```

which terminates the APL session.

A Practical Introduction to

APL 1&2

...programming, that is, the construction of a desired
function from a set of available primitive functions.
Ken Iverson

Day 2 : Second Generation – APL 2

APL 2 - Nested Arrays

Session 3

- System Functions & Variables
- Primitive Operators
- Expressions & Statements
- Order of Execution of Operators
- Loops v APL (S-Matrix) Thinking
- Enclose & Vector Notation

Session 4

- Each Operator
- Binding Strengths
- Empty Arrays
- Building Tools
- Supplied Workspaces
- Evolution of APL (1956-2004)

...the ability to translate into APL procedures of interest in your own profession...
Ken Iverson

LESSON 7

Some Useful System Functions

System functions are functions which perform a task outside the domain of the APL workspace. They can be called within lines of APL code, unlike system commands which can not be called in an APL program. Some of these system functions perform tasks that were originally only available through system commands. For example, ⎕*EX* **expunge**s names.

```
⎕EX 'NUMS'
```

does the same as

```
)ERASE NUMS
```

And

```
⎕NL 2
```

returns a character matrix (**name list**) that includes all global *class 2* names which are listed by

```
)VARS
```

- Try

```
⎕NL 2 3
```

The class of a name (**name class**) is returned by monadic ⎕*NC* .

A very powerful system function, called ⎕*FX*, **fixes** a new function defined by its character right argument.

- Try, for example,

```
⎕FX 2 9⍴'R←L IF C R←C/L    '
```

List all the functions in the workspace. Why has this function been called *IF*?

This system function (⎕*FX*) is one of the reasons why APL is said to be "self-conscious", and cannot, in general, be compiled. The **execute** function (⍎) is another metamathematical concept in the language. You can write *programs that write programs* whose properties may not be deducible at the compilation stage.

- Useful niladic system functions include

⎕*AI* which returns current **account information**,

⎕*TS* whose result contains the current system **time stamp**,

and

```
⎕WA
```

which evaluates the remaining **workspace available**.

A function that has been interrupted is said to be *suspended* on a certain line. The system function ⎕LC returns the **line count**, and therefore

```
→⎕LC
```

is often used to resume execution from the line at which execution has been halted.

- A favourite system function is (niladic) ⎕AV which returns the 256 element **atomic vector**. Try it. :-)

- Why is ⎕AV a niladic function and not a variable? Why does it have 256 elements?

System Variables. There are also a number of noteworthy *system variables*. They can be set (assigned) by the programmer and may have a profound effect on the results of APL expressions.

For example, the primitive function **interval** (⍳) returns an initial ordered subset of the natural numbers or a subset of the whole numbers depending upon the value of the **index origin** (⎕IO).

- Type in the following. Observe and explain the differences.

```
      ⎕IO←0
      ⍳9
0 1 2 3 4 5 6 7 8
      ?9
5
      ⎕IO←1
      ⍳9
1 2 3 4 5 6 7 8 9
      ?9
3
```

- Why does the following give the same answer whatever the **index origin**.

```
      'abcdefghij'[⍋⍒10⍴1 0]
afbgchdiej
```

- Write a simple APL model of some *fuzzy rule-based system*, };-)> basing your model on the conclusions you reach from the results of

```
      1<1.00000000001
1
      ⎕CT←1E¯10
      1<1.00000000001
0
```

LESSON 8

Primitive APL Operators

"By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems…," explains A.N.Whitehead. "The quantity of meaning compressed into a small space by algebraic signs .. facilitates the reasonings we are accustomed to carry on by their aid," observes Charles Babbage.

The ease of expressing constructs which arise in problems is one objective test that can be applied to a notation. We saw the difficulty we had in simply adding together all the elements in a vector. We now introduce the concept of an APL **operator** that solves this problem in a very powerful way. "An operator may be applied to a function to derive a different function," says the *IBM APL Language* manual of 1978.

General mathematical notation includes Σ to signify the sum of a series and Π to signify the product of a series. The linear APL notation introduces an operator **reduce** (/) which applies to a function, such as **plus** or **times**, to produce a derived function, such as sum or product.

```
      + / 1 2 3 4 5
15
```

This adds up all the numbers 1 to 5 by conceptually putting a + between all the elements. Thus +/ can be read as **plus reduce** and, in this case, is equivalent to

```
      1+2+3+4+5
15
```

Equally, by this definition of the derived function,

```
      × / 1 2 3 4 5
120
```

multiplies together all the numbers 1 to 5 by conceptually putting a × between all the elements. ×/ can be read as **times reduce** and is equivalent to

```
      1×2×3×4×5
120
```

In other words the mathematical statement $\sum_{i=1}^{n} X_i$ is equivalent to +/X in APL and $\prod_{i=1}^{n} X_i$ is ×/X .

- Sum the squares of the first 10 positive integers.

- Write a function which returns the largest number in a list.

Experiment with **reduce** operands ⌈ ⌊ - ÷ ∧ ∨ , giving arguments to the derived functions that are rank 2, 3 or above. Which *dimension* is "reduced"? Try to express some algebraic identities, such as (+/XρY) ≡ (Y×X) or (×/XρY) ≡ (Y*X)

A second operator introduced in core APL is **scan** (\). Like reduce, this operator applies to all dyadic scalar functions to produce a derived function that is conceptually similar to placing the dyadic scalar function between each of the elements along the last axis of the argument. However, **scan** generates all the intermediate values as well.

- Notice how

```
      +\1 2 3 4 5
1 3 6 10 15
```

gives all the (cumulative sum) results

```
1
1+2
1+2+3
1+2+3+4
1+2+3+4+5
```

- Explain the result of

```
      -\1 2 3 4 5
1 ¯1 2 ¯2 3
```

- Check the truth values of

```
∧/(+\XρY)=(Y×⍳X)
∧/(×\XρY)=(Y*⍳X)
```

- Experiment with **scan** applied to ⌈ ⌊ - ÷ and especially to ∧ ∨ ⍲ < ≤ ≠

Notice how an operator that takes only one (function or array) operand has it to the *left* of the operator symbol, while a function which has only one (array) argument has it placed on the *right* of the function symbol. This simple distinction increases the number of parsing rules in the core APL language from 1 to 2! (The second grammatical rule in APL is a bit like the rule in English that the subject of a sentence comes before the verb.)

Rule 2: The left function operand of an operator is the function derived from the longest possible operator sequence to its left.

It is not really possible until lesson 11 to understand the full force of this rule - until operators are used with derived function arguments. However, it is important to appreciate that *these two rules are 'essentially' all that is necessary to specify completely the parsing of any APL 1 statement*.

With the above two rules you should be able to read and understand (i.e. interpret) any rational first generation APL statement. This sentence is circular because by *rational* we mean any construct which adheres to rules 1 and 2 only. However, it is still worth saying because one very respectable goal of APL has been to discard all irrational constructs and replace them with rational ones. Unfortunately history has tended the other way – to add new arbitrary syntactic rules to second and third generation APL interpreters.

- Define a function that returns the average of a vector of numbers when the function argument is a vector, or the row averages when the argument is a matrix. Discuss how a primitive operator might be defined that specifies subarrays to which this *AVERAGE* function applies?

Operators may apply to *two operands* (c.f. arguments) but, unlike functions, they can NOT be *ambipotent* (c.f. ambivalent). An operator, exclusively, either takes one operand on its left or two operands, one on the left and one on the right (i.e. it is *monistic* xor *dualistic*). [Usually the words monadic, dyadic and valency are applied equally to functions and operators but here I introduce new words *monistic*, *dualistic* and *potency* to help the (newbie) APLer to clarify the important distinction between function and operator syntax.]

The **inner-product**, or the dot operator (.), requires two function arguments that combine in a given way to produce a dyadic derived function. In the case where the left operand is **plus** and the right operand is **times**, the derived function is that of *matrix multiplication* (which implicitly involves multiplication and plus reduction).

- Given some small test matrix,

```
MAT←3 3⍴⍳4
```

verify the results of

```
MAT+.×MAT
MAT+.×⍉MAT
MAT∧.=⍉MAT
MAT∧.=MAT
MAT⌈.+MAT
```

- In what context might each of the above derived functions be used? Consider, for example a statement such as

```
TOTAL_COST ← COSTS +.× QUANTITIES
```

In the early days of APL 1, operators were not well understood and a couple of syntactically irrational operator constructs were introduced. (See e.g. *Rationalised APL* by K.E.Iverson, 1981.) For example, the two symbols jot (∘) and dot are used in combination to represent the **outer- product** operator. This operator derives a dyadic function which conceptually takes each element of its left argument and combines it with each and every element in the right argument according to the dyadic (right) operand.

- Enter these two examples and find an example of another useful operand.

```
      (⍳7) ∘.+ (⍳7)        ⍝ addition table
2  3  4  5  6  7  8
3  4  5  6  7  8  9
4  5  6  7  8  9 10
5  6  7  8  9 10 11
6  7  8  9 10 11 12
7  8  9 10 11 12 13
8  9 10 11 12 13 14
```

```
      (⍳7) ∘.× (⍳7)      ⍝ multiplication table
 1  2  3  4  5  6  7
 2  4  6  8 10 12 14
 3  6  9 12 15 18 21
 4  8 12 16 20 24 28
 5 10 15 20 25 30 35
 6 12 18 24 30 36 42
 7 14 21 28 35 42 49
```

Notice the appearance of the *comment* symbol (⍝) which can be used to add comments at the end of any executable line of APL.

Aside: The jot-dot-times derived function, although irrational in its syntax, has been so admired by APLers that an American APL journal was named after it. There is also an American journal called APL Quote-Quad. Quote-quad is an I/O variable like quad (⎕) but for characters. It is written ⍞. Likewise there was an APL company called Inner Product and another called Dyadic Systems. Dyadic Systems had an Outer Products catalogue of end user solutions. Puns or what?! It is of interest to note that the name Dyalog originates from a joint project between Dyadic and Zilog whose outcome was the first version of the Dyalog APL interpreter in 1982 :-)

The **axis** operator has irregular syntax. It surrounds its numeric "right" operand with brackets. This, like indexing, does not follow the usual single token nomenclature which is assumed in rules 1 and 2. (But IBM APL2 and Dyalog now have a rational dyadic **index** function (⌷).)

- Experiment with

```
      ⌽[⎕IO]MAT
3 4 1
4 1 2
1 2 3
      ⌽[1+⎕IO]MAT
3 2 1
2 1 4
1 4 3
```

and

```
      MAT,[⎕IO]MAT
1 2 3
4 1 2
3 4 1
1 2 3
4 1 2
3 4 1
      MAT,[1+⎕IO]MAT
1 2 3 1 2 3
4 1 2 4 1 2
3 4 1 3 4 1
```

to see how the axis operator modifies the axis to which its left operand applies.

In the penultimate example above, the **catenate** function (**,**) becomes the derived function catenate-along-first-axis (**,** `[1]`) which can also be programmed as **catenate-first** (`⍪`) .

- Hence check the meaning of

```
      +/[1]MAT
8 7 6
```

and

```
      +/[2]MAT
6 7 8
```

- What might the following line of code be doing?

```
      ((+/[2]EXPENSES_BY_MONTH)>2500)/[1]SALESMEN
```

- Can you identify any other irrational syntax (i.e. irregular use of tokens vis-à-vis rules 1 and 2) in any of the preceding lessons? Is `/` a function or an operator?

Remember, you're not really supposed to encounter anything irrational in APL notation!

Grammar: Core APL, here called APL 1, has an extremely simple and elegant grammar. This grammar still forms the unchanged core of modern APLs, which adhere quite strictly to ISO Standard 8485:1989. Later generations, APL 2 and APL 3, each introduce a number of new rules of grammar making the underlying parsers considerably more complicated, and consequently, lines of code harder to read with confidence. Nevertheless, the original two grammatical rules of APL 1 still hold good and form a solid, clear foundation for the whole language. Fluency in APL follows from a secure understanding of the two fundamental rules of grammar stated in Lessons 1 and 8.

LESSON 9

APL Thinking

Most 20th century computer languages have evolved from representations of sequential instructions in digital memory. APL syntax, on the other hand, evolved from traditional mathematical notation. This explains the relative ease with which complex ideas can be translated into APL programs. As a result APL has been, and still remains, irreplaceable for many diverse mission-critical systems found in large organisations all over the world.

- Insert lots of redundant parentheses to show the steps involved in this statement:

```
SCALE ← 4 5 6 + 3 × ⍳ 9 - 6
```

- Construct the following matrix in the clearest possible way, and then again using as few syntactic elements (tokens) as possible:

  0 3 2 1
  1 0 3 2
  2 1 0 3

- Define a function which will remove any of a list of unwanted characters (the left argument) from a matrix (the right argument) and replace them with spaces. This is not a common utility but you've got to feel confident that you can quickly produce it if you need it. If it is destined to become a common utility then you should make sure that it is suitably general; it applies to all appropriate ranks of array arguments including zero rank and other *empty arrays*, it applies to numbers too, perhaps it even signals its own specific errors and it can never halt in the middle. In other words, it should be modelled on the design of primitive APL functions such as **member** (∈).

If you think that is too easy then show, instead, how to construct some interesting looking matrix with just a few tokens. If you think that is too simple as well, then generate all the whole numbers from 1 to 100 using any APL primitives but only using the digits 1,9,8 and 6 once each and in that order. Thus, for example, the number 1 is given by

```
      1⌊986
1
```

and the number 42 is given by

```
      19⌈⌈8*⊗6
42
```

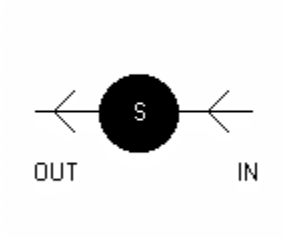
An amazing complete answer to this wonderful challenge is to be found in the fascinating and useful Journal of the British APL Association, in *Vector Vol.3 No.2 p105 (1986).*

Scalars & Loops versus Vector Operations. We have seen how it is possible to write a looping function that will add up all the elements in a vector. This should be replaced by the derived function **plus reduce** (or sum) for reasons of *both elegance and efficiency*. **Plus reduce** also extends naturally to sum over the rows, or sum over the planes by way of the **axis** operator (or by dyadic **transpose** (⍉), or by **reduce-first** (⌿) ...).

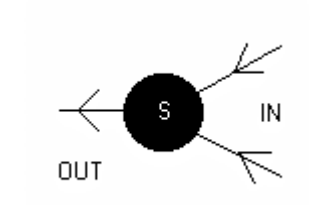
In APL one attempts to apply some function to the given arguments to obtain immediately the desired result. This is not always possible; often intermediate results are needed. If so, the problem is broken down into sub-problems and a solution to each sub-problem is then sought through the application of some function to an argument or arguments to give an intermediate result. It is amazing how seldom (if ever) loops are strictly necessary, even in core APL 1. (In APL 2, as we shall see, loops are employed even less often.)

Aside:
This *black-box*, *operational* way of thinking [input/operation/output] is reminiscent of the *scattering-matrix* approach to quantum mechanics which was popular amongst particle physicists in the 1970's (see *The Tao of Physics* by Fritjov Capra). One can observe what (particle) goes in and what (particle) comes out, but one can never see what "actually happens" inside a sufficiently small black circle. Mathematically, the (possibly infinite) matrix operation S simply transforms the (Hilbert space) vector IN to vector OUT. *Bootstrap philosophy* attempted to specify the S-matrix elements entirely through self-consistency with related S-matrices but then quantum chromo-dynamics (QCD) was discovered (c. 1973) which seems to describe accurately the behaviour of quarks and so theoretical physicists moved back to field theory thinking.

A monadic APL primitive function looks like S in:



A dyadic primitive function looks like S in:



This is a mental model of, or a template for, all APL programs (functions and operators): something precise goes *in*, and something precise comes *out*. No loops. No extraneous considerations. Just some encapsulated algorithm for transforming what goes in to what comes out – that algorithm is the APL program S.

Sometimes apparently unavoidable loops can be avoided by recursion. The Fibonacci series is elegantly generated through a recursive function. Each term in the series is the sum of the preceding two terms, starting with 1 1. The function below takes a whole number, *N*, and returns the first *N Fibonacci numbers*.

```
      ∇ Z←FIBONACCI N
[1]    ⍝ Z is the first N terms of the Fibonacci series
[2]     →(N=1)/'→~Z←1'
[3]     Z←Z,+/¯2↑Z←FIBONACCI N-1 ∇
```

- Study this function. Predict what will happen if N is zero. Verify that the quotient of the last 2 numbers of a finite Fibonacci series tends to the Golden Mean `((5*0.5)-1)÷2`

Hint: Remember Ctrl+C on a mainframe or Ctrl+Break on a PC!

Aside: The Golden Mean is believed by some to control psychology in the city? It appears that when a market price is ready for a big change, the next stable point downwards is the product with the Golden Mean, and the next stable point upwards is the quotient!

- Define a recursive *FACTORIAL* function which takes a whole number argument, *N*, and returns the **factorial** (!) of *N*, i.e. $N \times (N-1) \times (N-2) \times .. \times 2 \times 1$.

Hint: If N is not zero, then the result is `N×FACTORIAL(N-1)`.

- The following function converts a date to an International Day Number. Is the function *IDN_FROM_TS* millennium-proof? :-S

```
      ∇ IDN←IDN_FROM_TS TS;YY;MM;DD;LP;DDM
[1]    ⍝ Convert date from 3-element vector (3↑⎕TS) to IDN
[2]     →(0=+/TS)/'→IDN←0'
[3]     YY MM DD←TS             ⍝ Split args.
[4]     LP←-/0=400 100 4|YY     ⍝ Leap year *****
[5]     DDM←0 31,LP+59 90 120 151 181 212 243 273 304 334
[6]     IDN←⌊365.25×YY-1900     ⍝ Convert years.
[7]     IDN←IDN+1↑MM↓0,DDM      ⍝    ..    months.
[8]     IDN←IDN+DD-LP           ⍝    ..    days.
[9]     IDN←IDN+IDN<0           ⍝ Days before 1900-01-01 are ≤ 0.
      ∇
```

- Discuss the steps involved in programming the inverse function *TS_FROM_IDN*. ☹

Study the following remarkable function and suggest some better local variable names.

```
      ∇ R←EASTER YEAR;A;B;C;D;E;F;G;H;I;K;L;M;N;O;P
[1]    ⍝Returns the date of Easter in the specified YEAR(s)
[2]    ⍝Method from Butcher's Ecclesiastical Calendar (1876)
[3]    ⍝valid for all years in Gregorian calendar (1583 onwards)
[4]     A←19|YEAR ⋄ B←⌊YEAR÷100 ⋄ C←100|YEAR
[5]     D←⌊B÷4 ⋄ E←4|B ⋄ F←⌊(B+8)÷25 ⋄ G←⌊(B+(-F)+1)÷3
[6]     H←30|(19×A)+B+(-D)+(-G)+15
[7]     I←⌊C÷4 ⋄ K←4|C
[8]     L←7|32+(2×E)+(2×I)+(-H)+(-K)
[9]     M←⌊(A+(11×H)+22×L)÷451
[10]    N←H+L+(-7×M)+114
[11]    O←⌊N÷31 ⋄ P←31|N
[12]    R←(10000×YEAR)+1+P+100×O
[13]   ⍝∇ 23/07/1993 10:06:36 Z1STS3  ∇EASTER
[14]   ⍝∇⍝Converted from 44 UTIL by Ray Cannon
[15]   ⍝∇:EASTER 3 Jan 1996  16:38 C:\WDYALOG\WS\UTILS\UTIL44
      ∇
```

Knowing what a function's *name* is ought to help considerably in confirming one's understanding of the purpose of the function. A well chosen function name may even imply its valency and whether it returns a result or not. Careful choice of words and appropriate naming conventions should also apply to the names chosen for local and (particularly) global variables. For example, in traditional text books on Vector Analysis, vectors are written in bold or underlined letters to make them stand out, while in Einstein's more general tensor notation, the rank of an array is implied by the total number of subscripts and superscripts.

- Find the **matrix-inverse** of a random (non-singular) matrix and verify that it is indeed the inverse using **plus inner-product times**.

```
      ⌹?7 7⍴100
¯0.0013  ¯0.0024   0.0036  ¯0.013   0.015    0.014  ¯0.011
 0.011    0.0074  ¯0.0083   0.017  ¯0.019  ¯0.0059 ¯0.0014
 0.013    0.00039  0.0034  ¯0.011  ¯0.012  ¯0.0033  0.0081
 0.0047   0.0027  ¯0.015    0.0036 ¯0.0015 ¯0.0054  0.014
¯0.016    0.022    0.0041   0.024   0.0088 ¯0.0045 ¯0.028
 0.00094 ¯0.018    0.014   ¯0.018   0.011    0.0084  0.0013
¯0.011   ¯0.009   ¯0.00098 ¯0.0037  0.00043  0.0049  0.019
```

LESSON 10

Enclose & Vector Notation

Up until this lesson we have been learning APL 1. In the early 1980's, a number of APL vendors almost simultaneously introduced a new concept into APL - the *nested array*. Each element of a nested array can itself be any other (nested) array. A new monadic primitive function, called **enclose**, was introduced that took any array as its argument and returned a scalar *enclosed array*. This scalar (rank zero) array could then be inserted in any element of any APL array.

There were two inequivalent approaches; *grounded* arrays as proposed by Ken Iverson and implemented by I.P.Sharp Associates (SHARP APL), and *floating* arrays proposed by Jim Brown and implemented by Dyadic Systems (Dyalog APL), IBM (APL2) and STSC (APL*PLUS).

The essential difference between these two schemes is that in the grounded system, enclosing (<) a *simple scalar* produces an *enclosed scalar*, whereas in the floating system, enclosing (⊂) a simple scalar leaves it unchanged. We here follow the current mainstream floating array model of IBM's APL2 .

- Given the simple array

```
      A←3 3⍴⍳9
```

enclose it.

```
      ⊂A
1 2 3
4 5 6
7 8 9
```

Note that the display form is indented one character. Check the **rank** is that of a scalar.

- Now **assign** this scalar to the second row and second column position of A.

```
      A[2;2]←⊂A
```

- What is the **shape** of A now?

Data Structures. This opens up a whole new heterogeneous world!

- Execute the line below a few times and examine the structure of A each time.

```
      A[2;2]←⊂A
```

- **Reshape** (⍴) and **rotate** (⌽) a vector of vectors (a list of lists):

```
      VECVEC←'NOT' 'A' 'DRUM' 'WAS' 'HEARD'
```

Is `VECVEC` a vector of vectors?

Vector Notation.

Rule 3: A series of two or more adjacent expressions results in a vector whose elements are the *enclosed* arrays resulting from each expression.

(This is fortified with an eight level hierarchy of binding strengths. Grounded arrays introduced no new rules.)

Vector notation may be introduced as a generalisation of a simple numeric list because

```
      1 2 3
```

is the same as

```
      (1)(2)(3)
```

which is the same as

```
      (⊂1),(⊂2),(⊂3)
```

because enclosing a simple scalar leaves it as a simple scalar - unchanged.

- Use the supplied function *DISPLAY* (in workspace UTIL) to examine *VECVEC*, and *A*, and also:

```
      (⍳3)'ABCD'
 1 2 3  ABCD
      2 4⍴(⍳3)'ABCD'(2 2⍴0)
 1 2 3  ABCD  0 0    1 2 3
              0 0
 ABCD   0 0   1 2 3  ABCD
        0 0
```

- Primitive scalar functions *pervade* nested arrays. Interpret the results of expressions:

```
      ⌊.5 (2 3.7)
0  2 3
      (1(2 3))+(⊂4 5),⊂6,⊂7 8
5 6  8  10 11
```

- Many *structural* and *selection* primitive functions generalize naturally to nested arrays. Explain the results of

```
      'DEB' 'NORMA' 'PAT'⍳'DEB'
4 4 4
      'DEB' 'NORMA' 'PAT'⍳⊂'DEB'
1
      'DEB'∈'DEB' 'NORMA' 'PAT' ⋄ (⊂'DEB')∈'DEB' 'NORMA' 'PAT'
0 0 0
1
```

and give the dyadic epsilon function (∊) a suitable name, after having determined its part of speech given the grammatical context. ☺

Some new APL 2 primitive functions, for example monadic **disclose** (⊃), monadic **first** (↑) and dyadic **match** (≡), apply to nested arrays to give well defined results. IBM APL2 (and second generation SHARP APL) introduced complex numbers as an integral part of the language.

- Experiment with `0J1`, which means √-1, and `4J3` which means (4+3*i*).

```
      ¯1*0.5
0J1
      4+3×0J1
4J3
      +4J3
4J¯3
```

- Check the result of the complex addition and multiplication.

```
      2.1J3.4 1.4J6.2 + 4J3
6.1J6.4 5.4J9.2
      2.1J3.4 1.4J6.2 × 4J3
¯1.8J19.9 ¯13J29
```

- Explain the result of `⊗¯1` by way of the most beautiful and astonishing identity in mathematics:

$$e^{i\pi} = -1$$

LESSON 11

The Each Operator

Having introduced nested arrays, one immediately wishes to know how to apply primitive APL functions to them. Some primitive functions, such as **take** and **reverse**, have an obvious generalisation. The scalar functions are generalised to be *pervasive* which means that their action applies at all levels of nesting.

In order to apply an arbitrary function to each element of a nested argument, a new monistic operator is introduced in APL 2. The operator is called **each** (¨). It can take a monadic or dyadic function operand and return, respectively, a monadic or dyadic derived function. This derived function applies the operand separately to each element of a nested argument to return an element of the final result.

This definition has deep consequences, not least to obviate looping solutions on nested structures.

- Define a nested vector

```
      DATA←(5 (2 4))(14 (5 7))(9 19)100
```

- Copy and use the *DISPLAY* function or varChar to see the structure.

```
      DISPLAY DATA
.→-------------------------------------.
| .→--------. .→---------. .→---. |
| |    .→--. | |     .→--. | |9 19| 100 |
| | 5 |2 4| | | 14 |5 7| | '~---' |
| |    '~--' | |     '~--' | |
| '∈--------' '∈---------' |
'∈-------------------------------------'
```

- Use monadic **shape** to show that *DATA* is a vector of vectors (and scalars).

```
      ⍴DATA
4
      ⍴¨DATA
 2  2  2
```

- Try using the dyadic **reshape** function on **each** element of *DATA*.

```
      3 ⍴ ¨ DATA
 5  2 4  5  14  5 7  14  9 19 9  100 100 100
```

- Why is the result the same as

```
      (4⍴3) ⍴¨ DATA
 5  2 4  5  14  5 7  14  9 19 9  100 100 100
```

- How many lines of code are there in all the functions in your workspace?

Hint: Consider the shape of each of `⎕CR¨⊂[2]⎕NL 3`

- Notice and understand the differences between

```
      ⌽DATA
100   9 19   14   5 7    5   2 4
```

and

```
      ⌽¨DATA
  2 4   5    5 7   14   19 9   100
```

and

```
      ⌽⍤⍤DATA
 5   4 2    14   7 5    9 19   100
```

- What will be the results of

```
      4 5 3 4↑¨DATA
```

and

```
      1 2 1 1↓¨DATA
```

Hint: What is the shape of `1↓0` *or* `1↓'A'`*? Ouch! Oooo!* `:-O`

In APL, scalars are rank zero arrays and as such they fit very neatly into the whole APL data scheme. Thus the shape of a numeric *scalar* is a vector of length **zero**, such as `0⍴0`. The shape of a *vector* (rank one) is a vector of length **one**, such as `,5`. The shape of a *matrix* (rank two) is a vector of length **two**, such as `10 10`. The shape of a rank three *array* is a vector of length **three**, such as `3 4 5`. etc…

- Predict the result of, and simplify, expression

```
      2⍴(0⍴0)⍴0
```

- Investigate the results of

```
      3⍴¨(⍳8)2(⌽⍳6)
 1 2 3   2 2 2   6 5 4
      3⊃¨(⍳8)(⍳7)(⌽⍳6)
3 3 4
      2↑¨,¨(2 2⍴5)(3 3⍴6)
 5 5   6 6
      ⍴⍤⍤⍤⍤⍤⊂(2 3)(2 2⍴⍳4)(⊂3⍴4)
      3
```

- Build nested data structures

```
      3 5⍴'a' 1 'abc' 1 2 3(1 2 3)
a       1   abc       1     2
3   1 2 3     a       1   abc
1       2     3   1 2 3     a
      2 3⍴'Briony' 25 'Kings Furlong' 'Karen' 27 'Winklebury'
 Briony   25  Kings Furlong
 Karen    27  Winklebury
```

- Experiment with the following monadic and dyadic derived functions. Feel the extra power. ☺

```
      ⊂⍤⍤

      ⍴⍤⍤

      ⍋⍤⍤

      ⍒⍤⍤
```

- Show that the solutions to the sets of simultaneous equations

$$\begin{aligned} x + y &= 3 \\ x - y &= 2 \end{aligned} \qquad \text{and} \qquad \begin{aligned} 2x + 3y + z &= 3 \\ 2x + y - z &= 2 \\ x - y - z &= 1 \end{aligned}$$

are given by

```
      (3 2)(3 2 1)⌹⍤(2 2⍴1 1,1 ¯1)(3 3⍴2 3 1,2 1 ¯1,1 ¯1 ¯1)
 2.5 0.5  1.5 ¯0.25 0.75
```

LESSON 12

APL Idioms

There are very many idiomatic expressions in APL. There are, for example, *hundreds* of possible derived functions produced from the inner product operator alone, some of which are idiomatic.

Many expressions are particularly useful and appear again and again in everyday code. Here is a small opening selection from the very comprehensive list of **Finnish APL Idioms** which contains over 700 entries of gradually more complicated expressions. (Alpha (⍺) represents the left argument and omega (⍵) represents the right argument of the implied function.)

```
Rank of array ⍵ ..................................................  ⍴⍴⍵
First one (turn off all ones after first one) ....................  <\⍵
Not first zero (turn on all zeroes after first zero) .............  ≤\⍵
Alternating product ..............................................  ÷/⍵
Any, anyone ......................................................  ∨/⍵
Parity ...........................................................  ≠/⍵
All, both ........................................................  ∧/⍵
Ascending cardinal numbers (ranking, all different) ..............  ⍋⍋⍵
Grade up according to key ⍺ ......................................  ⍋⍺⍳⍵
Test if integer ..................................................  ⍵=⌊⍵
All axes of array ⍵ ..............................................  ⍳⍴⍴⍵
Sorting ⍺ according to ⍵ .........................................  ⍺[⍋⍵]
Sorting ⍵ into descending order ..................................  ⍵[⍒⍵]
Index of (first) maximum element of ⍵ ............................  ⍵⍳⌈/⍵
Conversion from characters to numeric codes ......................  ⎕AV⍳⍵
Maximum of ⍵ with weights ⍺ ......................................  ⍺⌈.×⍵
Sum of squares of ⍵ ..............................................  ⍵+.*2
Test if ⍵ is within range ⍺[1] to ⍺[2] ...........................  ⍵<.≤⍺
Test if rows of ⍵ contain elements differing from ⍺ ..............  ⍵∨.≠⍺
Joining pairs of ones ............................................  ⍵∨≠\⍵
Vector having as many ones as ⍵ has rows .........................  ∨/0/⍵
Test if ⍵ and ⍺ have elements in common ..........................  ∨/⍵∈⍺
Resistance of parallel resistors .................................  ÷+/÷⍵
Scan from end with function ⍺ ....................................  ⌽⍺\⌽⍵
Replace elements of ⍵ not satisfying ⍺ with blanks/zeroes ........  ⍺\⍺/⍵
Indices of ones in logical vector ⍵ ..............................  ⍵/⍳⍴⍵
Number of columns in matrix ⍵ ....................................  ¯1↑⍴⍵
```

- Why might `'⊞'[1+2|Z∘.+Z←⍳⍵]` be called “chessboard” when ⍵ is 8?

- And why might `' .*'[1+⍵∘.≥((⍳⍺)÷⍺)×⌈/⍵]` be called “histogram”? Try it when ⍺ is 50 and ⍵ is `?⍳99` .

- Why was `⍵[⍋⍒(⍴⍵)⍴0 1]` called “the perfect shuffle” by the gurus at I.P.Sharp?

- Can you invent two other contenders for the FinnAPL list? Think specific and applicable. Remember to consider the possibility of arguments of different ranks. Think general and beautiful. Pay particular attention to the form of results from *empty* arguments - arguments whose shape contains zeros. Use the APL primitive functions as exemplary atomic models for user-defined utility functions. You are now well on your way to being a valuable APL programmer.

LESSON 13

Building Tools

Good advice to the APLer is ***Build Tools***.

Gather together, or build from scratch, a set of utility functions which are modelled on the APL primitive functions. Let them do one specific job with argument arrays of any suitable rank and type. Make sure they have no external impact apart from the results they return (e.g. no change to global or system variables). These utilities, if well chosen, can form the basic capsules for each and every new application which you write.

The utility below *right justifies* a numeric or character vector.

```
      ∇ VEC←RJUST VEC
[1]     VEC←(-+/∧\⌽VEC=1↑0⍴VEC)⌽VEC
      ∇
```

- Write a function `LJUST` which *left justifies* all the rows in a matrix.

Hint: Do this in two distinct steps.

- Write a function which trims the leading (and trailing) spaces from a character vector. This function is very useful. What might you consider when choosing an appropriate function name?

- Write a function `UCASE` which efficiently converts nested character arrays to *upper case* letters.

- Write a function, `AFTER`, which returns a boolean scalar signifying whether or not the left argument (date) is temporaneously after the right argument (date). `:)`

- Write scalar dyadic functions, `LCM` and `HCF`, which return the lowest common multiple - the smallest number which is exactly divisible by each of two numbers - and the highest common factor (or greatest common divisor) - the largest number which divides each exactly.

- When writing an APL function, which of the following suggestions should you adopt?

1) Choose meaningful names in the application context, not single letters.
2) Make functions as short as possible – less than 15 lines long and less than 60 characters per line.
3) Make programs as fast as possible to run rather than as easy as possible to read.
4) If you can't avoid global variables, use distinct names; for example, `DATA` rather than `DATA`.
5) Use line labels like `Step2:` rather than `L2:` and never branch to absolute line numbers.
6) Don't put more than one assignment arrow in a statement and avoid loops.
7) Explain the arguments, purpose and result of all functions in a comment at the beginning.
8) APL standards are a matter of individual taste and style and should not be imposed.

- Discuss the APL programming standards which *you* intend to follow.

LESSON 14

Investigating Supplied Workspaces

Every APL interpreter comes with a treasure trove of supplied workspaces. Some are collections of utilities, some are small example applications and some are full-blown applications.

- Enter the system command (possibly followed a library name or number)

```
)LIB
```

to see the list of workspaces available in your own library, or in a public library.

Warning: Before loading any new workspace, `)SAVE` *your work.*

- Find a workspace which sounds interesting and type `)LOAD` followed by the name of the workspace (possibly preceded by a library number or directory path).

When you load a workspace the contents of the system variable `⎕LX` is executed. Thus loading a workspace may start a whole application.

- If you have started an application, hit the interrupt key (perhaps a second or third time - sometimes also hit the enter key a couple of times!) to break the execution. (PCs are somewhat more predictable than mainframes in this regard.) When the program has stopped, type

```
)VARS
)FNS
)SI
```

The State Indicator resulting from the `)SI` command when a function is suspended shows the calling structure and the line on which execution halted.

- Use your editor to open the function at the top of the stack, and identify the line on which execution is suspended.

- A line is *restartable* if it can be run again and again without changing the outcome. Look carefully at the suspended line to determine if the line is restartable. If so, type

```
→⎕LC
```

otherwise try to restart from an earlier (`→⎕LC-1`) or later (`→⎕LC+1`) line or load another workspace.

Finally, to exit APL, type

```
)OFF
```

LESSON 15

The Evolution of APL

This is an exciting time for APL. APL might seem old - it started to emerge in Iverson's postgraduate thesis on machine solutions to linear differential equations at Harvard in 1956 – but it is only in the last two decades that APL has made close contact with the world of traditional computing. In the 1980's, APL.68000 and APL*PLUS were early, somewhat solitary, inhabitants of microprocessors. Prior to that, APL was mainframe-specific and very idiosyncratic.

APL 1. APL was first implemented using FORTRAN on an IBM 7090 in 1964. Before this crucial advance the notation was used for teaching and for formal descriptions only. The first timeshare version was implemented in 1965 and made generally available from 1968 on IBM System/360. From 1969 I.P.Sharp offered a commercial APL timesharing service which extended to Europe in 1973. In 1973 IBM announced APLSV which had shared variable support.

APL 2. Second generation IBM APL2 appeared in 1982 based on the 1971 doctoral thesis of J.A.Brown at Syracuse University. In 1981, when Ken Iverson left IBM and joined I.P.Sharp Associates, I.P.Sharp released a more rigorous, but incompatible, implementation of SHARP APL for handling arrays of arrays. In 1981 the I.P.Sharp network expanded to the Far East. In mid 80's STSC and Dyadic Systems released second generation APLs compatible with IBM APL2. Another significant development for APL throughout the 80's was the mass move from mainframe to micro. STSC APL*PLUS/PC under DOS on the IBM (compatible) PC proved a particularly popular combination.

APL 3. In the 90's APL and the Microsoft Windows operating system became friends. Windows 3.0 released in 1990 opened up the world of the Graphical User Interface (GUI) to Dyalog APL/W, APL*PLUS II, APL.68000 Level II, J and other dialects of APL. Another related technology of the 1990's which has been exploited extensively by Dyalog APL/W is Object Linking and Embedding (OLE). These two interfaces offered by Microsoft (GUI and OLE) have given APL a clear route by which to communicate with the Windows operating system and hence with other PC applications.

APL 4. The new century has placed computing on a distributed Internet platform where APL can compete directly with other popular computer languages. This new superhighway has a well-defined and accessible set of communication protocols and information formats (TCP/IP, HTTP, XML…). When writing and running APL programs on the Internet is as easy as writing and running them in an ordinary APL session then the full power of APL will at last become visible, not just to experienced APL programmers and those large corporations who have the bespoke APL applications, but to everyone with a need to powerfully manipulate information. The special APL character set has been an obstacle to APL accessibility and acceptability. This particular obstacle will be completely removed by the full introduction of UNICODE fonts. Further, Microsoft Vista purports to replace entirely the Windows Application Programming Interface (API) with the new .NET Framework which Dyalog APL/W Version 10 can already utilize in XP.

APL thinking could turn the Internet from a closed driven system like Microsoft Word, Excel or IE6 into an open programming environment like immediate execution mode - from the current incredible Internet HTML slide show to a vast knowledge engineering base in which world-wide information is distilled and understood.

One of many examples of the powerful inroads that APL is making onto the new information superhighway is found in an article by Jonathan Barman in the journal of the British APL Association. This article describes a utility function which translates an arbitrary APL array into an XML character string. XML (as well as UNICODE no doubt) has been called *"the ASCII of the future"*.

```
XML ← Encode Array
```

See: J.Barman in Vector Vol 17.4, April 2001, page 48.

- Begin work on a simple application of APL in a topic in your own discipline.
- Go and solve real problems…
- Consider attending further courses on different aspects of APL, such as:

A Practical Introduction to

APL 3&4

with Dyalog APL

Day 1: Third Generation – APL 3 – GUI & OLE

Using GUI Objects (⎕WC)
Object Properties & Methods
Object Events & Callback Functions (⎕DQ)
Looking inside the APL Session (⎕SE)
Understanding Dot Syntax
Arrays of Namespaces
Getting into Word & Excel from APL (OLE)
Writing In-Process OLE Servers (.DLL)
Writing ActiveX Controls (.OCX)
How to Access Functions in .DLLs (⎕NA)
Control Structures
Making Runtime .EXEs

Day 2: Fourth Generation – APL 4 – TCP/IP & .NET

Dynamic Functions & Operators
Introduction to MultiThreading
Aspects of Pocket APL
Understanding TCP/IP Sockets
Browsing the Internet from APL
Hosting an APL Web Server
Introduction to Dyalog .NET
Writing .NET Classes
Writing Web Services
Writing ASP.NET Web Pages
Writing Dyalog APL Classes

FURTHER RESEARCH

This 2-day course is just an introduction to the big, wide world of APL programming. For current information on APL you should subscribe to *Vector* magazine (the Journal of the British APL Association), and visit the Vector web site at

http://www.vector.org.uk/

Or subscribe to *APL Quote Quad* and visit the American special interest group web site at

http://www.acm.org/sigapl/

Many other APL organizations and links may be found in, for example,

http://www.acm.org/sigs/sigapl/orgs.htm

or

http://www.rexswain.com/aplinfo.html

or

http://www.chilton.com/~jimw/others.html

or

http://www.lescasse.com/TSPX5.asp

For an example of a modern successful APL application, see

http://www.grapl.com/

There is also a steady stream of miscellaneous opinions, questions and answers about APL in the open Google comp.lang.apl group to be found at

http://groups.google.com/groups?hl=en&lr=&ie=UTF-8&group=comp.lang.apl

* * * * *

F E E D B A C K F O R M

Date _____ Instructor _____

Name _____ Course Name _____

eMail _____ Location _____

Please indicate your view of the following:

poor 1 2 3 4 5 excellent

☐ ☐ ☐ ☐ ☐	Location & Facilities
☐ ☐ ☐ ☐ ☐	Course Content
☐ ☐ ☐ ☐ ☐	Course Material
☐ ☐ ☐ ☐ ☐	Instructor's Knowledge
☐ ☐ ☐ ☐ ☐	How useful was the course to your role?

Could any improvements be made to the course?

Comments

Give this form to your tutor, or send to ROBERTSON (Publishing).

EXERCISE 1

1. Write an expression that will surround a character matrix, `MAT`, with quad characters (`⎕`).

2. Convert the result of `⎕TS` into a date of the form dd/mm/yyyy.

3. Write an expression that returns the volume, V, of a sphere of radius R, given that $V = (4/3)\,\pi\,R^3$. Does your expression hold for a set of spheres of radii R_i?

4. Reversibly, encode a character matrix into numeric form.

5. Write a function that converts a character string to upper case.

6. Write a function that takes a numeric matrix and returns a vector of column averages.

7. Write a function that removes the leading and trailing spaces from a character string.

8. Write a function that replaces multiple spaces in a character string with a single space.

9. Write a function that creates a matrix from a character string in which an embedded carriage return character (`⎕AV[4]`) is used to identify the end of a line.

10. Write a function that creates a matrix from a character string in which embedded carriage return and linefeed characters (`⎕AV[4 3]`) are used together to identify the end of a line.

11. Write expressions to reproduce the results of *signum* (monadic `×`), and *residue* (monadic `|`).

12. Write an expression to find the value of a polynomial of degree (`⍴C`) with `C+1` arbitrary coefficients.

13. Find the coordinates of the centre of gravity (R_{CofG}) of a system of 10 particles given a 10 element vector of masses (M_i), a 10 by 3 matrix of positions (R_i) and the formula

$$R_{CofG} = \frac{\sum M_i \times R_i}{\sum M_i} .$$

14. Solve the system of linear equations for x, y and z

$$\begin{aligned} 3x - 2y + 4z &= 1 \\ 5x + 2y - z &= 2 \\ x - 4y - z &= 3 \end{aligned}$$

using matrix division (`⌹`).

15. Find an expression for the sum of the diagonal elements (the trace) of a numeric matrix.

16. In what sense is APL a computer language that has been 200 years in the debugging?

EXERCISE 2

1. What is the full path name and size in bytes of your Dyalog APL interpreter executable? What is the version number and build ID of this interpreter?

2. What is meant by scalar extension? What is the rank of a scalar? Are these concepts to be found in traditional mathematics?

3. Find by experiment the size of an empty workspace (.DWS file).

Hint: Use `)SAVE`.

4. Give one good reason why system commands begin with a right parenthesis. Is there any good reason why system variables and system functions all begin with a quad symbol?

5. Find by experiment the size of an empty APL component file (.DCF file).

Hint: Use `⎕FCREATE`.

6. Write a function that takes the full path name of a file and returns just the file name alone.

7. Write a function to surround a full path file name with double quotes (") in the event that this name contains spaces. (This is sometimes necessary, *eg* in `⎕CMD'dir "C:\program files"'`.)

8. How would you identify the current user from inside an APL program? Investigate the purpose of [Options][Configure][Network][Network ID] by finding a reference to APLNID in the User Guide *C:\Program Files\Dyalog\Dyalog APL 11.0\manuals\Dyalog APL Windows User Guide.v11.0.pdf* and in the Registry under section HKEY_CURRENT_USER\Software\Dyalog\Dyalog APL/W 11.0.

Hint: [Start][Run] REGEDIT.

9. Explore the meaning of the MAXWS parameter. Examine in Notepad the contents of the file to which the APLK parameter refers – keep this file open for the next question (**10.**).

Hint: Find the keyboard file name in the registry or in [Options][Configure][Keyboard].

10. Load the UTIL workspace and run the niladic function `KEYPRESS`. Press any key on the keyboard and relate the description of your KeyPress Event to the corresponding entry in your keyboard file.

11. Use the Workspace Explorer (via [Tools][Explorer] or the corresponding toolbar button) to investigate some of the elementary functions such as `CENTRE` and `MAKEMAT` in the UTIL workspace.

12. Write a monadic function that returns 1 if the character string in its argument is a valid name of a function in the current workspace, and 0 otherwise.

13. Interpret the operator sequence on line 6 of the function `SETMON` in the UTIL workspace, *viz* `(0∘,)∘⍳∘⊃∘⍴∘⎕CR⍤0`.

14. Put *zilde* (`⍬`) in the first component of an APL component file and put a depth 6 nested array in component 2. Untie the file and look at the structure of its contents using varChar (char.exe).

15. Write a recursive function to calculate the determinant of a matrix.

16. Write an operator which takes a scalar Boolean left operand and a monadic function right operand and applies the function to the argument of the derived function iff the left operand is 'true' (*ie* 1).

EXERCISE 3

1. What are the **major APL applications** currently in everyday use within your company?

2. Briefly describe the **purpose and function** of each. *Ask someone if you are not sure.*

3. For each of these applications, in which **directory** in your company's file system is each of the current **production**, **test** and **development** APL **programs** to be found? *Consult the experts if in doubt.*

4. In which **directory** is the corresponding system (read/write) **data** to be found?
Ask: remember half of an APL programmer's job is to eke out of somebody exactly what's to be coded.

5. Create a **3D array** of character vectors containing the above **directory** names. The items of the first dimension are APL *application names*, the 3 items of the second dimension are *promotion levels* **production**, **test** and **development**, and the 2 items of the third dimension are *application elements* **program** and **data**. What use might be made of this nested array? Can you think of possible 4^{th} and 5^{th} dimensions?

6. List **all the files** associated with the test version of one of your company's principle Dyalog APL systems and briefly explain the rôle or significance of each file. Use ***varChar*** to explore the contents of component (or native) **files**. Use your Dyalog APL development environment to discover where in the **workspace** each of these files is referenced.

7. Describe the **process** by which an APL workspace is promoted from development to test, and from test to production, within your company. How would you number and classify the various **versions** of a large critical APL application as it grows and evolves over days, months and years?

8. How might you ensure that an APL application is **thoroughly tested**? What information should you save in the event of a **program failure**? How is it possible to retrieve unsaved information and clues as to the cause after a Dyalog APL **interpreter crash**?

9. Describe how you might organise and administer programming tasks in the circumstance that an APL system is being developed contemporaneously by **a number of APL programmers**. Consider, for example, workspace organisation, function edit locking, programming style, task specification packaging, test and sign off procedures and data sharing "dimensions". Itemise any other "dimensions" that you deem worthy of consideration when managing a large APL team project.

10. Select one particular APL application of special interest and **trace** through the code at a more or less superficial level. Draw a simple diagram containing various significant program strands discovered in the code structure. Identify in the diagram points at which input and output screens are created, and places where alternative routes through the code may be freely chosen or predetermined.

EXERCISE 4

1. Write a looping function that encapsulates the Babylonian method of finding the square root of a real number, given that the Babylonian method is:

1. Make a guess
2. Divide the original number by the guess
3. Find the average of these two numbers
4. Use the average as the next best guess.

2. Convert your looping function into a recursive function.

3. Use APL to find all the prime numbers less than 10 thousand by way of the sieve of Eratosthanes:

1. Take all the integers from 2 to 10,000
2. Pick out the first integer in the list
3. Remove all multiples of this number from the list
4. Repeat 2 and 3 until the list is empty.

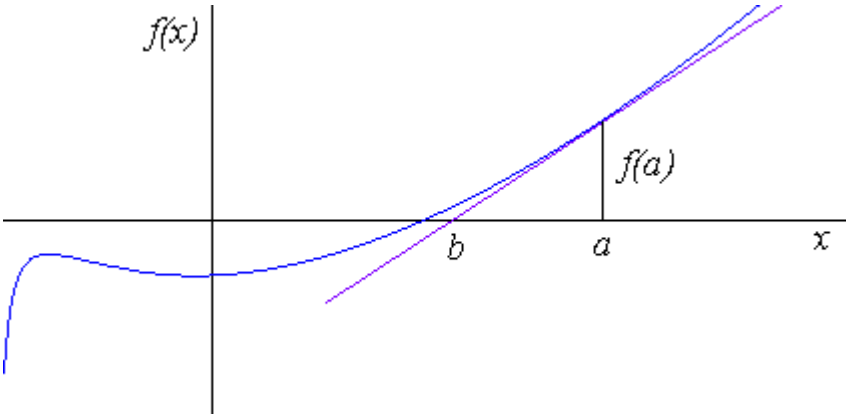
Compare your solution with that in the supplied dfns.dws workspace.

4. Find from the Taylor series

$$f(x) = f(a) + \frac{(x-a)}{1!}\frac{df}{dx}(a) + \frac{(x-a)^2}{2!}\frac{d^2f}{dx^2}(a) + \frac{(x-a)^3}{3!}\frac{d^3f}{dx^3}(a) + .. + \frac{(x-a)^n}{n!}\frac{d^nf}{dx^n}(a) + ...$$

an approximate expression for the derivative of a function at *a* in the event that second and higher derivatives may be ignored at point *a*. Write an APL operator whose derived function is the approximate derivative of the operand.

5. If *f(x)*=0, and *a* is a good first guess at the value of *x* when *f(x)*=0, then find, from the approximation above, a better second guess, *b*, for the value of *x*. Write a function that repeats this procedure a number of times (Newton-Raphson iteration) in order to get a more and more precise estimate of *x* when *f(x)*=0.



6. Find by experiment the approximate length of the longest possible variable *name* in Dyalog APL.

Hint: Use *execute* (⍎) and ⎕*WA*.

7. Interpret Conway's game of ***life*** as

```
↑1 BMat∨.∧3 4=+/,¯1 0 1∘.⊖¯1 0 1∘.⌽⊂BMat
```

and hence use ⍣ to play

```
life life life 5 8⍴40↑0 0 1 0 0 0 0 0 1 0 1 0 0 0 0 0 0 1 1
```

EXERCISE 5

1. Write a function that takes an integer argument `w`, and returns a vector *x* of `w` equally spaced (*dx*) numbers between ¯5 and 5.

Big hint: `x←((-w+1)+⍳1+2×w)÷w÷5 ⋄ dx←x[2]-x[1]`

2. The Gaussian function

$$\rho(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

maps values $x \in \mathbb{R}$ into the interval $0 < \rho(x) \leq (2\pi)^{-½}$. $\rho(x)$ may be interpreted in terms of the probability of obtaining $x \in X$, given that X is a random variable that is **normally distributed** with mean value 0, standard deviation 1 and total probability 1. Translate this function into APL and verify that

$$\int_{-5}^{5} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \approx 1.$$

Hint: `+/dx×(÷(○2)*0.5)××-(x*2)÷2`

3. Write an operator ***E*** which takes an operand of a probability distribution $\rho(x)$ and a vector argument *x*, and returns the **expectation** of *x* under this distribution, given that mathematically

$$E(x) = \int_{-\infty}^{\infty} x\, \rho(x)\, dx\,.$$

Note that the vector *x* should be sorted before *dx* can be calculated.

Consider: `+/dx×x×(÷(○2)*0.5)××-(x*2)÷2`

4. Write an operator ***Var*** which returns the **variance** of a vector of numbers, given that mathematically

$$Var(x) = E((x - E(x))^2)\,.$$

Consider: `E(⍵-¨E ⍵)∘.×(⍵-¨E ⍵)`

5. The APL primitive monadic function ***roll*** (`?`) takes an argument of any positive integer array, `X`, and returns a result that contains integers, pseudo-randomly selected from the sets of integers `⍳¨X` with each integer in each population having an equal (uniform) chance of being selected.

Write a monadic function called ***U*** that takes a positive integer vector argument (corresponding to the shape of some numeric array) and returns a vector containing half the number of elements in the original numeric array composed of a **uniform distribution** of numbers between 0 and 1.

Big hint: `U←{(?(⌈(×/⍵)÷2)⍴¯1+2*31)÷¯1+2*31}`

6. In 1958, Box and Müller published an algorithm for transforming a uniform distribution of random numbers into a **normal distribution** of random numbers by inverting the Gaussian function continued into $\mathbb{C}$:

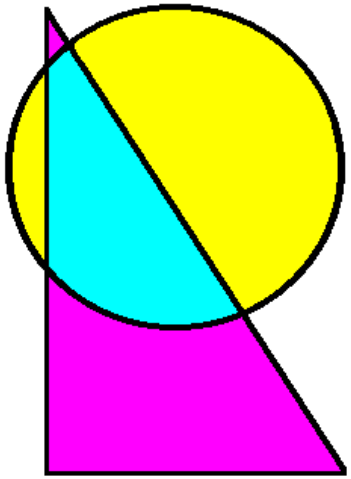
$$x_1 + ix_2 = \sqrt{-2\ln(y_1)}\; e^{2\pi i y_2}\,.$$

This takes 2 real numbers y_1 and y_2 from independent uniform distributions and yields 2 numbers x_1 and x_2 from independent normal distributions.

Write a monadic function called ***N*** which takes a positive integer argument and returns that many numbers normally distributed around 0.

Big hint: `N←{⍵⍴↑(⊂(¯2×⍟U ⍵)*0.5)×¨1 2○¨⊂○2×U ⍵}`

Certificate of Achievement



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